REPORT ON THE WORKSHOP: *INFORMATION THEORY AND NON-EQUILIBRIUM THERMODYNAMICS BEYOND THE SHANNON-GIBBS FRAMEWORK* VIENNA COMPLEXITY SCIENCE HUB, JANUARY 25TH-26TH 2018

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The main question leading all the discussions was *Is there a clear thermodynamic interpretation of generalised forms of entropy?*

Ramifications from this general question concern, among other, i) relations between computability, complexity and thermodynamics, ii) the use of generalised forms of entropies to refine predictions in classic physical problems, iii) the potential dual structure of the theory beyond equilibrium or iv) how to make sense of the zoo of functionals arising when we relax/modify the conditions that must be satisfied by an entropy. The workshop put together leading experts from different fields of the current research in out-of-equilibrium systems. This included researchers from information geometry, stochastic thermodynamics, computational complexity theory or generalised statistical mechanics, among others. The discussions were extremely interesting and it was made clear that there is still a long path to follow before the understanding of how generalised forms of statistical mechanics or information theory relate to energy flows. Below we detail the main contributions of the different participants.

- **Jan Naudts** revised recent developments of information geometry. This highly mathematical field gives a geometrical background to information theory and can elegantly rephrase all the results from classical statistical mechanics in the language of geometry. An important issue addressed in his talk was the dual nature of the thermodynamic theory, which is clearly established in equilibrium and provides a clear relation between intensive and extensive variables. Within the information geometry framework, he showed us that the dual structure exhibited by the exponential family –which we could link, at least informally, with the equilibrium hypothesis– could be recovered in much more general cases. These results could impact in the development of a much more general thermodynamic-like theory, since they demonstrate that the dual nature of the theory is not a property exclusive of equilibrium systems.
- **Petr Jizba** exposed the last developments to better ground and bound Heisenberg's uncertainty principle. He revised the differences between *variance uncertainty relations* (VUR) and *information theoretic uncertainty relations* (ITUR). The ITUR provide non-linear approaches to the uncertainty on the measurement, being able to achieve a finer resolution, thereby improving the bounds for the uncertainty relations. He showed us how the

use of entropy powers based on generalised forms of entropy –in this case, Rényi entropy– can even improve the bounds predicted by Shannon-based ITUR's. This use of a non-Shannon entropy to predict quantitative, testable bounds gave us a valuable example for the understanding of how these entropies may play a role in real, testable systems.

- **Karoline Wiessner** provided a clear classification of the problems of the so-called complex systems between non-equilibrium thermodynamics, complexity theory, information theory and its generalised forms. She revised the notions of complexity related to the so called ϵ-*machines* and how they link to thermodynamic cost. The approach presented by her was radically different –but also clearly complementary– from the two presented above, in the sense that the notion of complexity and how this relates to energy costs is not statistical in nature but based on the *machine* that creates the statistical patterns. These complexity-based approaches are of crucial relevance for the understanding of complex systems, where the structure and internal rules, beyond the statistical patterns, play a role which must have a thermodynamical pay-off.
- **Artemy Kolchinsky** gave a comprehensive review of the booming field of stochastic thermodynamics. In this approach, Shannon entropy and the Kullback-Leibler divergence play a grounding role, and relations between information and thermodynamics, like the Landauer's principle, arise naturally. In addition, the celebrated fluctuating theorems from Jarzynsky and others give exact thermodynamic relations between work, entropy production and free energies for states arbitrarily far away from equilibrium. He presented his own results on energy/work and information equivalences outside the equilibrium setting, and raised the question if the whole picture of stochastic thermodynamics could be grounded on different entropic functionals. Interestingly, he discriminated the options for generalisation based on f-*divergences* and *Breckman divergences*, exploring the conditions required by one or other generalizations, and observing that only the standard Kullback-Leibler divergence satisfies all the requirements demanded by both approaches.
- **Rudolf Hanel** presented a novel and promising approach in which the entropic functional is constructed directly from the underlying microscopic dynamics. In this framework, the microscopic ansatz is made explicit. The roots of this approach reach early Boltzmann's works, in which it is emphasised the validity of the statistical mechanics approach only under what Boltzmann called *strong chaos hypothesis*, under which one recovers all the standard functionals of Shannon information theory. Outside this case, things get unexpectedly involved. Taking the example of the recently introduced Sample Space Reducing processes or Pólya urns as a microscopical ansatz, in the talk he showed how the different faces of entropy –entropy production, phase space volume and max ent functional– unfold into three different functional forms. The unfolding of the entropy into different functional forms whenever the underlying dynamics is not totally uncorrelated opens important and unexpected question marks to the connection between information theory and the energy costs outside equilibrium.

- **Jan Korbel** invited us to join him into the search of the *Holy Grial* that would tell us what is the right entropy. He started reviewing recent achievements concerning the axiomatic approach to non-extensive systems. This axiomatic approach basically removes the fourth Shannon-Khinchin axiom –violated for most interacting systems. The presence of this axiom ensures uniqueness of the Shannon entropy. Based on a well known general gamma-like form of entropy arising from the three first Khinchin axioms, he presented a formalism able to finely disentangle different paradigms of phase space behaviour, from power-law growing to super-exponential. Further, he presented the functionals arising from the Shore Johnson axioms and related them to the inference problem of max-ent prediction. The proposal concluded with the need to merge the two approaches, namely, the behaviour of the phase space and the max-ent functional arising from statistical inference. Interestingly, the general entropic forms derived from this formalism satisfy both the requirements from the first three Khinchin axioms –therefore, valid to encode information about the phase space behaviour– and the Shore Johnson ones –therefore, able to satisfy the requirements imposed by statistical inference.

After the presentations of all the speakers, often accompanied by nice debates, we approached the plenary discussion.

The first conclusion, pointed out by almost all of the participants, was the enormous heterogeneity that characterised the different approaches. These different viewpoints affect not only the goals and the perspective taken within different subfields, but also the meaning itself of the involved information theoretic terms. This feature, undoubtedly, makes any effort of unification/cohesive interpretation enormously difficult. As an example, we observe that i) one can relate energy terms from the field of stochastic thermodynamics to information, even outside equilibrium. In addition, ii) the approach based on machine complexity can also provide bounds energetic bounds to information processing which are not directly tied to equilibrium. Nevertheless, both approaches i) and ii) are difficult to merge, since the information-theoretic concepts involved are subtly different. The second observation was that, outside the equilibrium setting, the unified theoretical framework seems to split into seemingly unbounded different formal subframes, depending on the particular assumptions of one or other approach. As an example, we find i) the search of the *right* entropy depending either on statistical inference or phase space considerations or ii) the splitting of the entropy in at least three different functional forms outside the multinomial-Bernoulli framework. This introduces an additional difficulty to the task of attributing energetic considerations to the generalised forms of statistical mechanics. Finally, the possibility or impossibility of the quantitative test was also discussed. Besides the relevant example provided by the information theoretical uncertainty relations based on entropy powers of the Rényi entropy, it was acknowledged that the generalised statistical mechanics setting has a gap in terms of quantitative/testable predictions.

After acknowledging all the mentioned difficulties, it was pointed out that it could be worth to explore the theoretical merging of some aspects of Information geometry with some others from stochastic thermodynamics. The reasons were i) the great mathematical consistency provided from the information geometry approach and ii) the capacity of stochastic thermodynamics to link energy related concepts arbitrarily away from the equilibrium. Other particular details of such a potential approach were discussed.

In spite of the seemingly negative outcome, the workshop was considered unanimously as a success. The reason is that it made clear what are the gaps between the different approaches. In addition, it showed us that maybe outside the standard setting, the beautiful unified picture of thermodynamics and information provided by Boltzmann, Gibbs, Landauer or Jaynes, among others, is completely broken and –maybe– it cannot be recovered in an analogous way in a more general case.

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