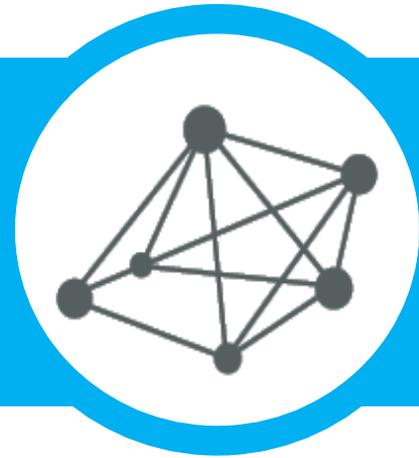




# SPHERA

## Controlling networks while maintaining resilience

Baruch Barzel





# Challenges



## 2003 NORTHEAST BLACKOUT

$5.5 \times 10^7$  People affected  
 $10^2$  Fatalities  
 $6 \times 10^9$  USD in damages



# Structure vs. dynamics

Structural  
perturbation  
(component failure)

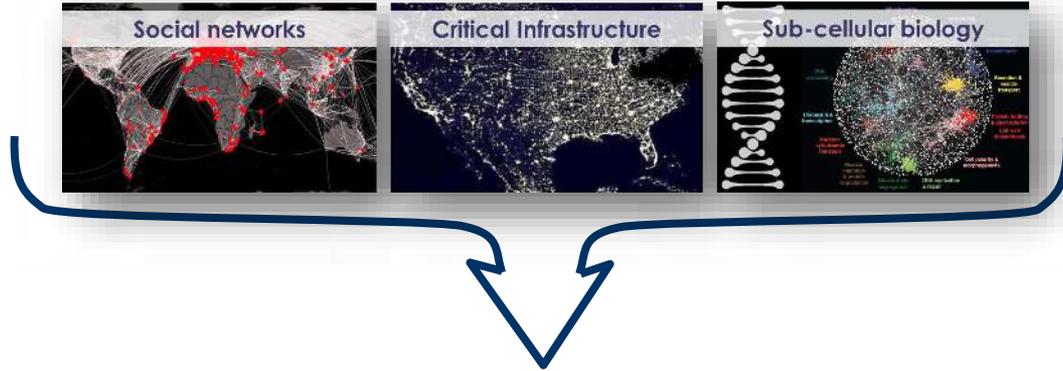


Can we predict the  
point of Resilience  
loss?

Dynamic outcome  
(Resilience loss)



# Dynamic framework



$$\frac{dx_i}{dt} = F(x_i(t), \boldsymbol{\varphi}_i) + \sum_{j=1}^N A_{ij} Q(x_i(t), x_j(t), \boldsymbol{\theta}_{ij})$$

$x_i(t)$  **State of a system component (node)**

- Concentration of a protein/metabolite
- Probability of infection of an individual
- Load on power/communications component



# Dynamic framework

$$\frac{dx_i}{dt} = F_i(x_i(t), \boldsymbol{\varphi}_i) + \sum_{j=1}^N A_{ij} Q_{ij}(x_i(t), x_j(t), \boldsymbol{\theta}_{ij})$$

## Interaction mechanisms



For example (gene regulation)

$$\frac{dx_i}{dt} = -C_i x_i^{\beta_i} + \sum_{j=1}^N A_{ij} \frac{x_j^{\alpha_{ij}}}{k_{ij} + x_j^{\alpha_{ij}}}$$

$F$

Self dynamics

$Q$

Interaction mechanisms

$\boldsymbol{\varphi}_i, \boldsymbol{\theta}_{ij}$

Distributed parameters



# Dynamic framework

$$\frac{dx_i}{dt} = F(x_i(t), \boldsymbol{\varphi}_i) + \sum_{j=1}^N A_{ij} Q(x_i(t), x_j(t), \boldsymbol{\theta}_{ij})$$

## Interaction mechanisms



Example: population dynamics

$$\frac{dx_i}{dt} = x_i \left( 1 - \frac{x_i}{C_i} \right) + \sum_{j=1}^N A_{ij} \frac{x_i x_j^{\alpha_{ij}}}{k_{ij} + x_i x_j^{\alpha_{ij}}}$$

## Network structure



$A_{ij}$



# Dynamic framework

$$\frac{dx_i}{dt} = F(x_i(t), \varphi_i) + \sum_{j=1}^N A_{ij} Q(x_i(t), x_j(t), \theta_{ij})$$

## Interaction mechanisms



- Nonlinear
- Multi-parametric ( $\varphi_i, \theta_{ij}$ )
- Black-box:  $F_i, Q_{ij}$  sometimes unknown

## Network structure



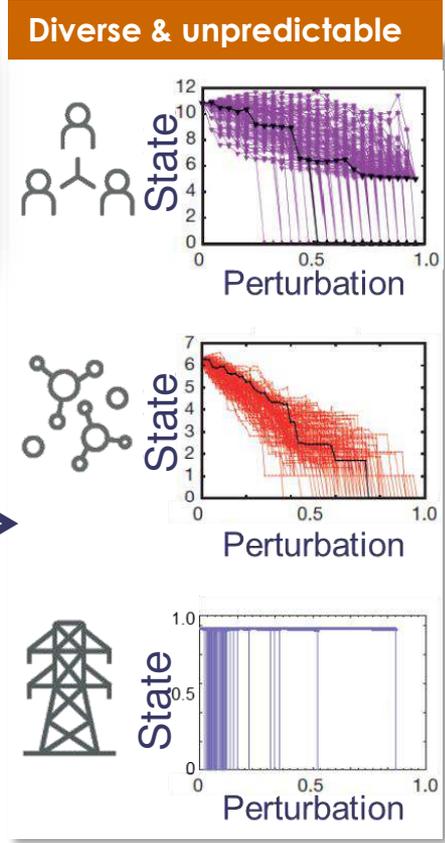
Weighted  
Heterogeneous (Scale-free)



# Diverse and unpredictable



Can we predict the point of Resilience loss?



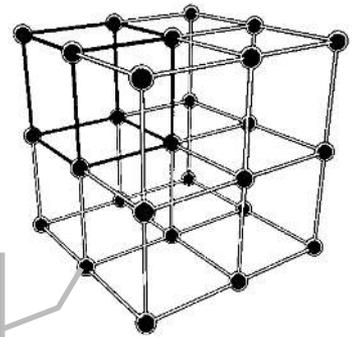


# A physicist's nightmare

## Current nonlinear dynamics theory:

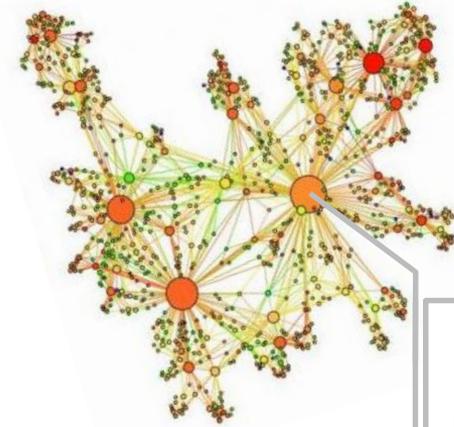


Each node has  $k = 6$  nearest neighbors



- Low dimensional
- Symmetric structures (lattice or lattice-like)

## Where real networks are:



$k$  spans orders of magnitude

- Disordered and weighted
- Extremely heterogeneous
- Scale free:  $P(k) \sim k^{-\gamma}$

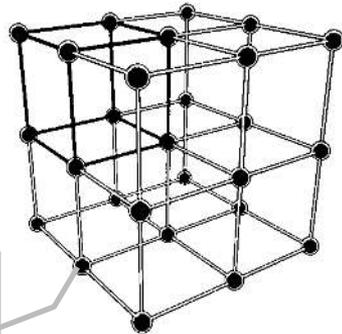


# Symmetry

## Zero order symmetry

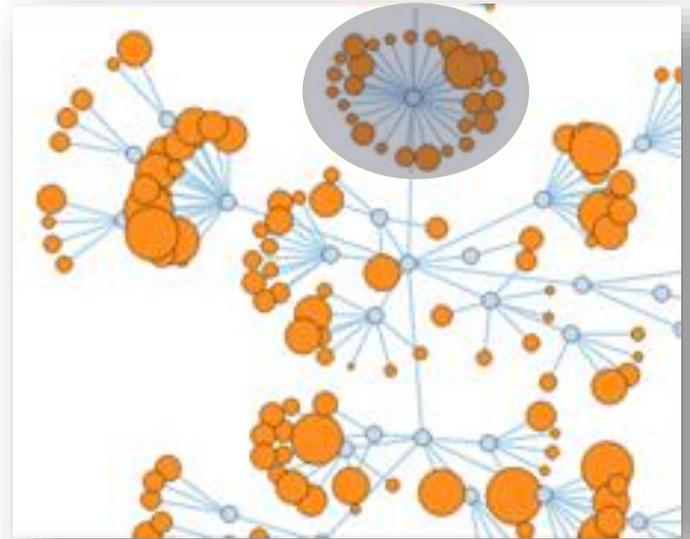


Each node has  
 $k = 6$  nearest  
neighbors



All nodes identical

## $n$ -order symmetry



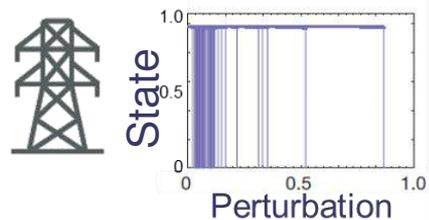
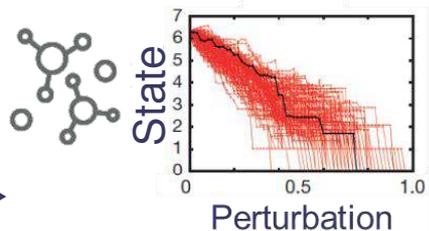
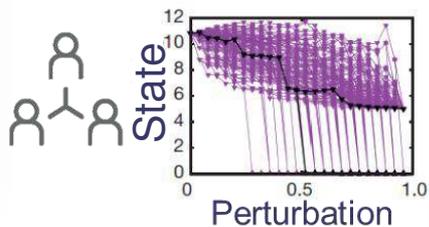
All environments identical

# Global control parameter



Can we predict the point of Resilience loss?

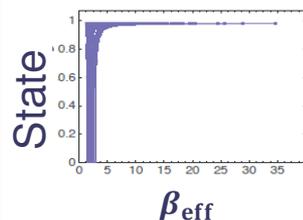
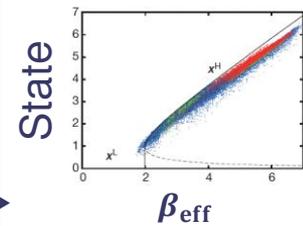
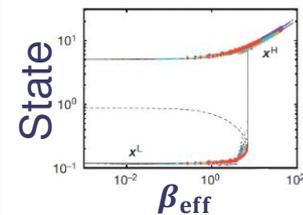
## Diverse & unpredictable



**Universal parameter**  
 $\beta_{\text{eff}}$  universally predicts the critical transition points of resilience loss

$$\beta_{\text{eff}} = \frac{\mathbf{1}^\top \mathbf{A}^2 \mathbf{1}}{\mathbf{1}^\top \mathbf{A} \mathbf{1}}$$

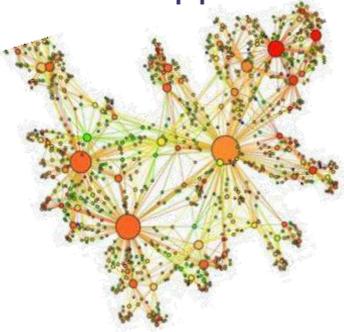
## Universal



# Global control parameter

## Structure $A_{ij}$

Well mapped



## Control parameter

$$\beta_{\text{eff}} = \frac{\mathbf{1}^\top A^2 \mathbf{1}}{\mathbf{1}^\top A \mathbf{1}}$$

Translating **Structure** into **Dynamic observables** of interest

## Resilience

A Dynamic observable of the system that we seek to predict, understand and influence

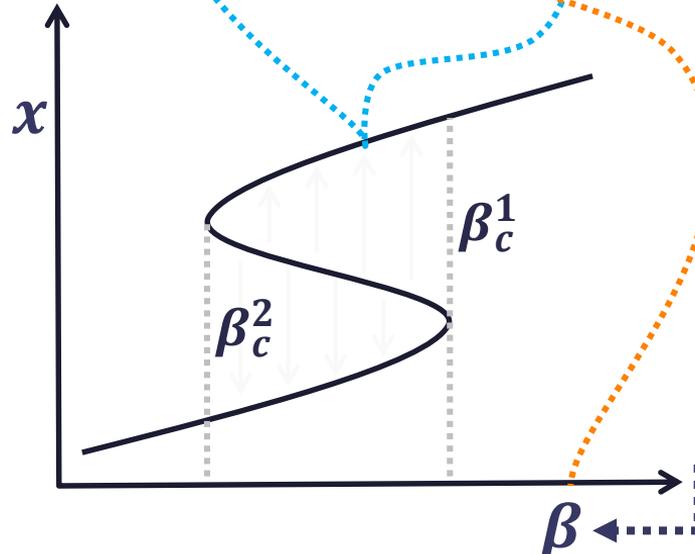


**Example:** Using the **Structure** of the power network to determine its **Dynamic resilience** against local failures or load perturbations



# Top-down - Global control parameter

$$\frac{dx_i}{dt} = F(x_i(t), \varphi_i) + \sum_{j=1}^N A_{ij} Q(x_i(t), x_j(t), \theta_{ij})$$



Bridges between  
**Topology** and **Dynamics**  
Sets guidelines for  
intervention

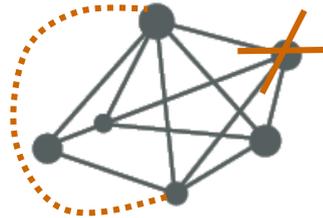


# Bottom-up - Intervention

$$\frac{dx_i}{dt} = F_i(x_i(t), \boldsymbol{\varphi}_i) + \sum_{j=1}^N [A_{ij} Q_{ij}(x_i(t), x_j(t), \boldsymbol{\theta}_{ij}) + B_{ij} S_j(t)]$$

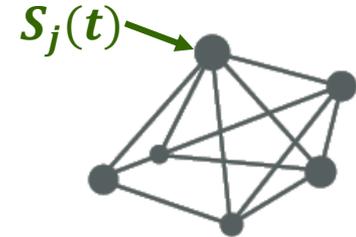
## Structural interventions

Removing nodes, adding links, changing weights



## Dynamic interventions

External signals  $S_j(t)$  to selected nodes



## Functional interventions

Manipulating  $F_i$  and  $Q_{ij}$  or their parameters





# Spatio-temporal spreading patterns

$A_{ij}$

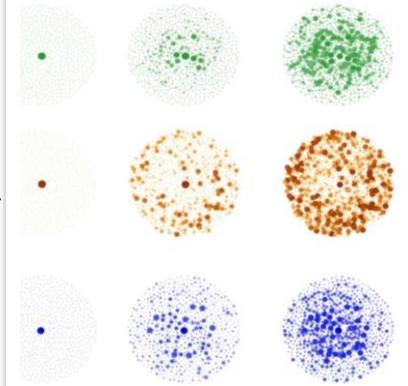


$F_i(x_i, \varphi_i)$

$Q_{ij}(x_i, x_j, \theta_{ij})$



Diverse & unpredictable

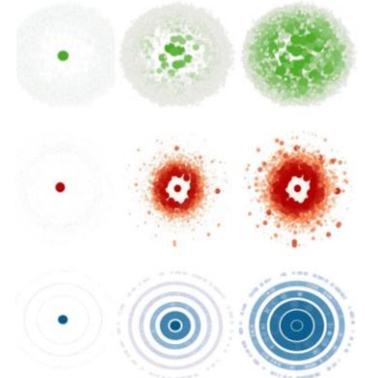


Time

Control parameter  $\theta$   
Individual node response time

$$\tau_i \sim k_i^\theta$$

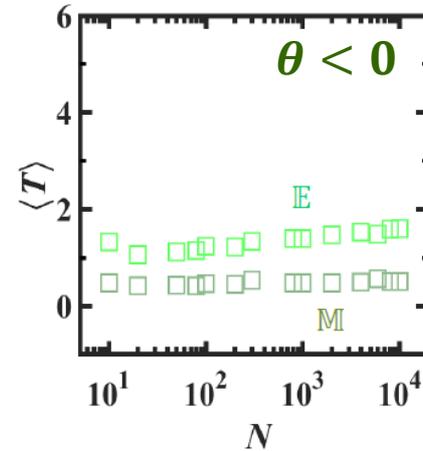
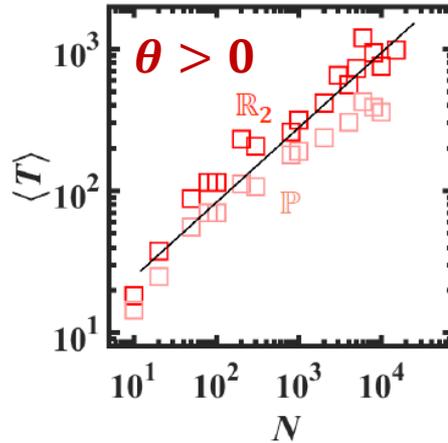
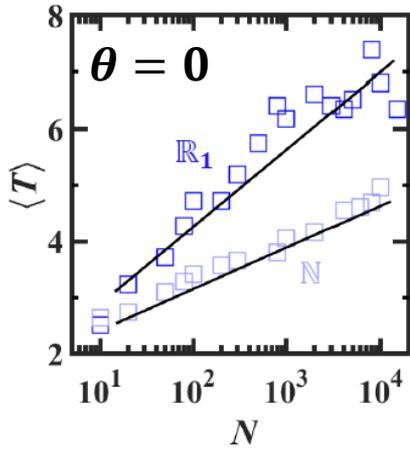
Universal



Time



# Soft stability

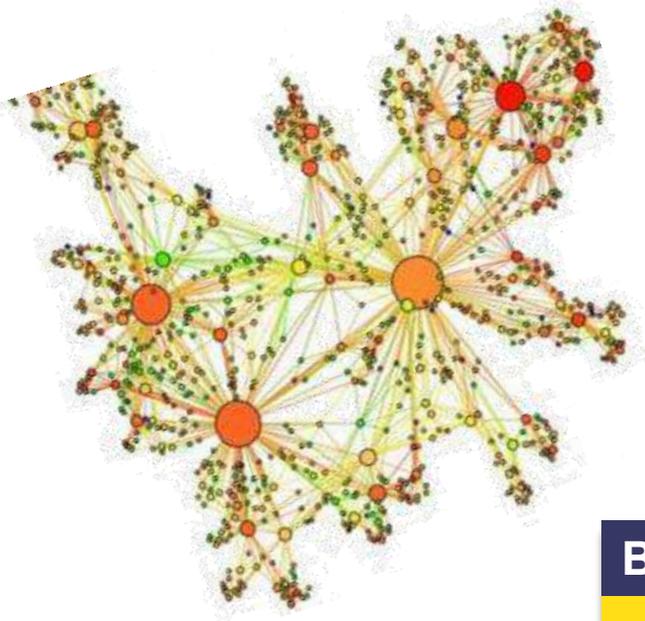


## Beyond stability

How much time do we have before an undesired transition occurs



# Optimizing functionality vs. resilience



## Beyond stability

How much time do we have before an undesired transition occurs



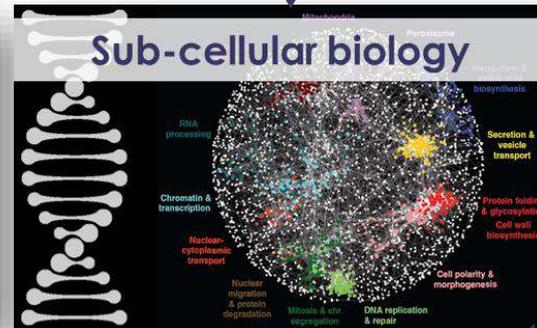
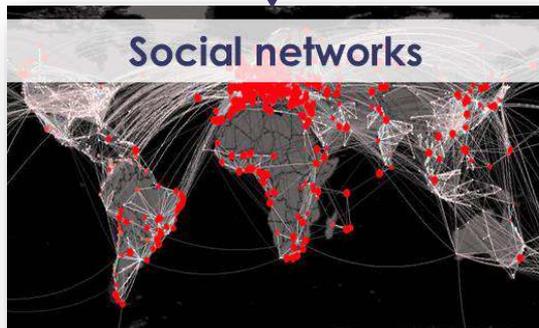
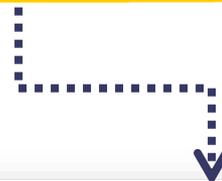
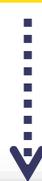
# Open threads

**Top-down** Identify macroscopic control parameters ( $\beta, \theta$ )

**Bottom-up** Selecting nodes for intervention (real-time mitigation)

**Stability vs. resilience** Enriching the discussion on stability

**Functionality vs. resilience** Can we introduce balanced incentives



In the top-down approach systems are influenced by means of global control parameters. Quite often these act as boundary conditions for the system dynamics. To identify such control parameters is a challenge on its own. Often they can be derived from the known macroscopic, or system dynamics. As a major conceptual drawback, control parameters usually reflect limitations of stability, rather than of resilience.

- In the bottom-up approach systems are influenced by specifically targeting some of the system elements, e.g. agents in an agent-based model or nodes in a network representation. Again, two different possibilities exist: (i) the agents can be controlled in their internal dynamics, or (ii) the agent interactions can be controlled.

Can we identify general principles for the bottom-up control of socio-economic or ecological systems? How can driver nodes be identified based on data-driven methods?

Can we explain the breakdown of resilience in social organizations as a misallocation of resources? What is the relation between resilience and the natural tendency of systems to maximize their performance?