



SPHERA

Controlling networks while maintaining resilience

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Challenges



2003 NORTHEAST BLACKOUT

5.5×10^7 People affected
 10^2 Fatalities
 6×10^9 USD in damages



Structure vs. dynamics

Structural
perturbation
(component failure)

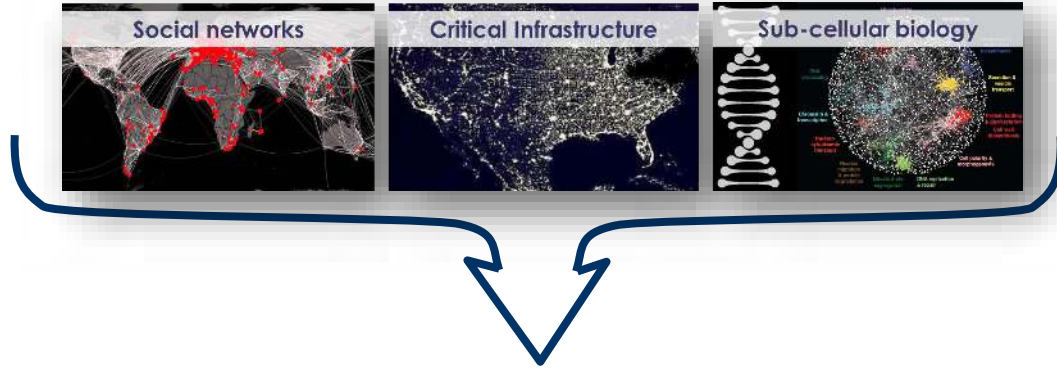


Can we predict the
point of Resilience
loss?

Dynamic outcome
(Resilience loss)



Dynamic framework



$$\frac{dx_i}{dt} = F(x_i(t), \boldsymbol{\varphi}_i) + \sum_{j=1}^N A_{ij} Q(x_i(t), x_j(t), \boldsymbol{\theta}_{ij})$$

$x_i(t)$ State of a system component (node)

- Concentration of a protein/metabolite
- Probability of infection of an individual
- Load on power/communications component



Dynamic framework

$$\frac{dx_i}{dt} = F_i(x_i(t), \boldsymbol{\varphi}_i) + \sum_{j=1}^N A_{ij} Q_{ij}(x_i(t), x_j(t), \boldsymbol{\theta}_{ij})$$

Interaction mechanisms



For example (gene regulation)

$$\frac{dx_i}{dt} = -C_i x_i^{\beta_i} + \sum_{j=1}^N A_{ij} \frac{x_j^{\alpha_{ij}}}{k_{ij} + x_j^{\alpha_{ij}}}$$

F

Self dynamics

Q

Interaction mechanisms

$\boldsymbol{\varphi}_i, \boldsymbol{\theta}_{ij}$

Distributed parameters



Dynamic framework

$$\frac{dx_i}{dt} = F(x_i(t), \boldsymbol{\varphi}_i) + \sum_{j=1}^N A_{ij} Q(x_i(t), x_j(t), \boldsymbol{\theta}_{ij})$$

Interaction mechanisms



Example: population dynamics

$$\frac{dx_i}{dt} = x_i \left(1 - \frac{x_i}{C_i} \right) + \sum_{j=1}^N A_{ij} \frac{x_i x_j^{\alpha_{ij}}}{k_{ij} + x_i x_j^{\alpha_{ij}}}$$

Network structure



A_{ij}



Dynamic framework

$$\frac{dx_i}{dt} = F(x_i(t), \varphi_i) + \sum_{j=1}^N A_{ij} Q(x_i(t), x_j(t), \theta_{ij})$$

Interaction mechanisms



- Nonlinear
- Multi-parametric (φ_i, θ_{ij})
- Black-box: F_i, Q_{ij} sometimes unknown

Network structure



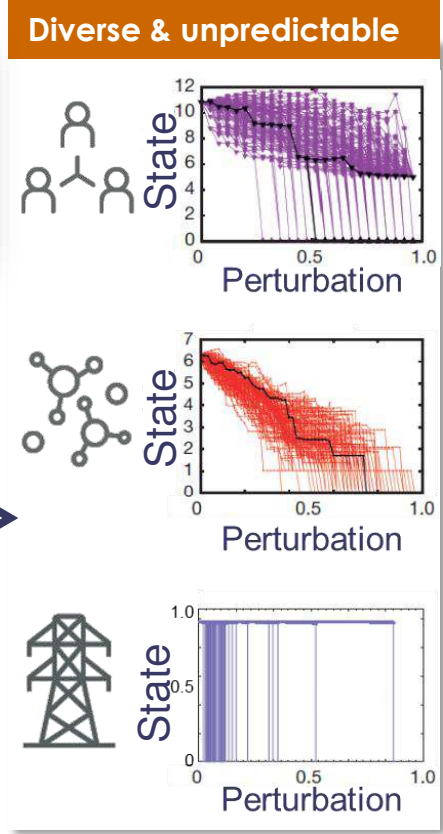
Weighted
Heterogeneous (Scale-free)



Diverse and unpredictable



Can we predict the point of Resilience loss?



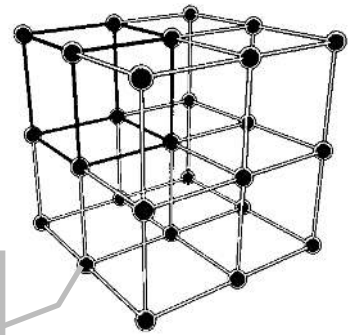


A physicist's nightmare

Current nonlinear dynamics theory:

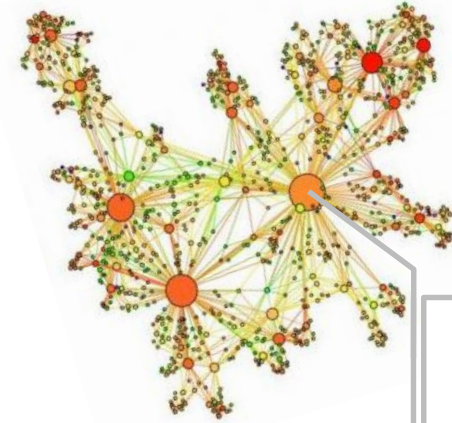


Each node has $k = 6$ nearest neighbors



- Low dimensional
- Symmetric structures (lattice or lattice-like)

Where real networks are:



k spans orders of magnitude

- Disordered and weighted
- Extremely heterogeneous
- Scale free: $P(k) \sim k^{-\gamma}$

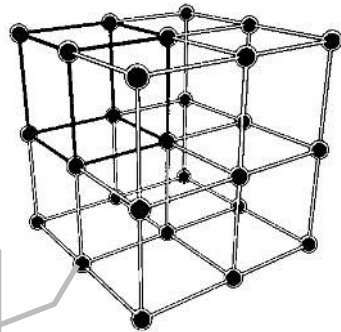


Symmetry

Zero order symmetry

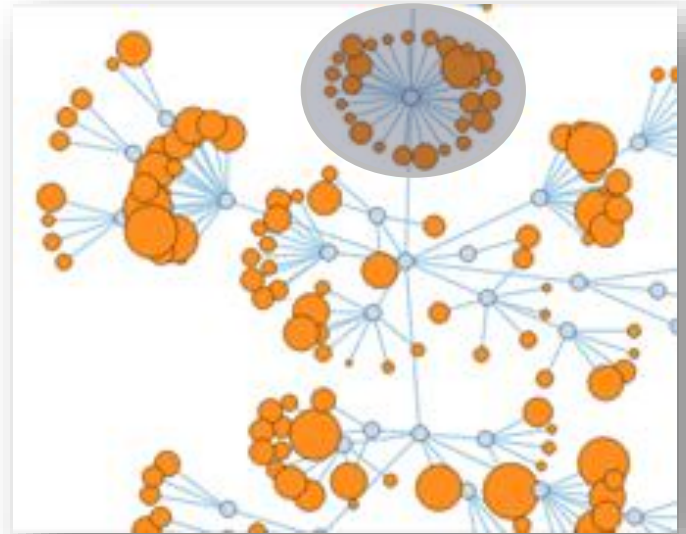


Each node has
 $k = 6$ nearest
neighbors



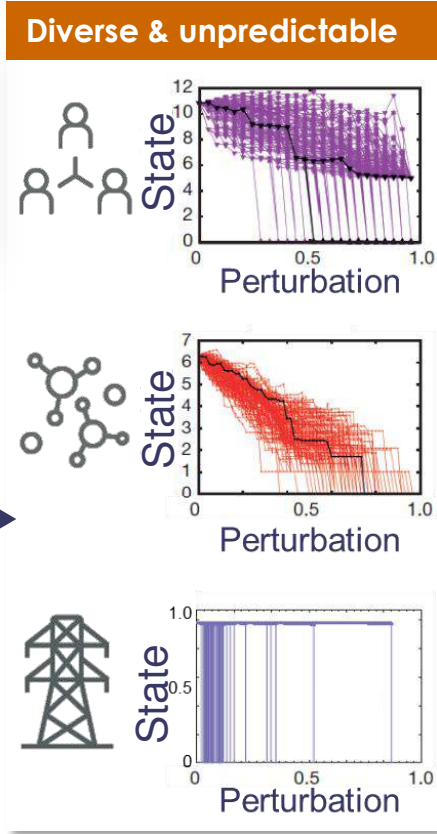
All nodes identical

n -order symmetry



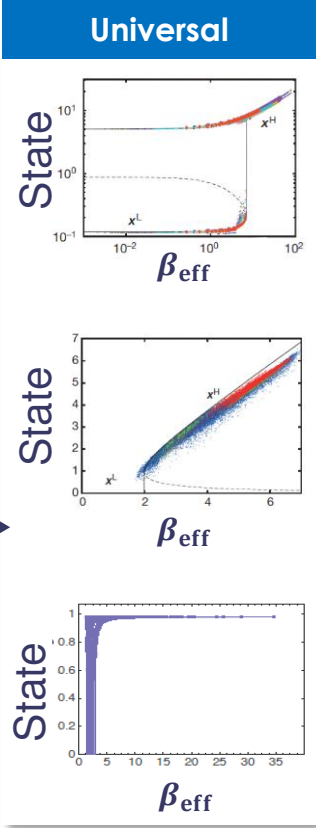
All environments identical

Global control parameter



Universal parameter
 β_{eff} universally predicts the critical transition points of resilience loss

$$\beta_{\text{eff}} = \frac{\mathbf{1}^\top \mathbf{A}^2 \mathbf{1}}{\mathbf{1}^\top \mathbf{A} \mathbf{1}}$$



Global control parameter

Structure A_{ij}

Well mapped



Control parameter

$$\beta_{\text{eff}} = \frac{\mathbf{1}^\top A^2 \mathbf{1}}{\mathbf{1}^\top A \mathbf{1}}$$

Translating **Structure** into **Dynamic observables** of interest

Resilience

A Dynamic observable of the system that we seek to predict, understand and influence

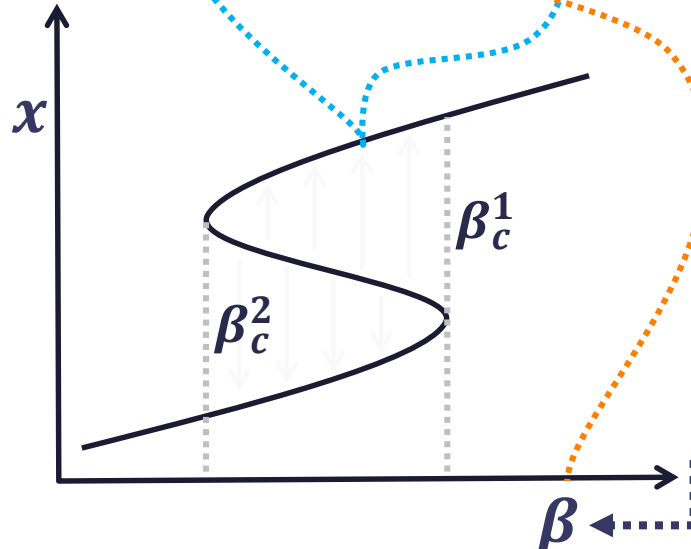


Example: Using the **Structure** of the power network to determine its **Dynamic resilience** against local failures or load perturbations



Top-down - Global control parameter

$$\frac{dx_i}{dt} = F(x_i(t), \varphi_i) + \sum_{j=1}^N A_{ij} Q(x_i(t), x_j(t), \theta_{ij})$$



Bridges between
Topology and **Dynamics**
Sets guidelines for
intervention

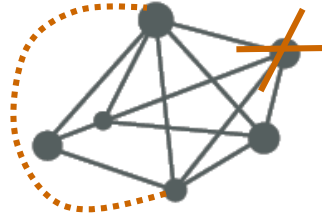


Bottom-up - Intervention

$$\frac{dx_i}{dt} = F_i(x_i(t), \boldsymbol{\varphi}_i) + \sum_{j=1}^N [A_{ij} Q_{ij}(x_i(t), x_j(t), \boldsymbol{\theta}_{ij}) + B_{ij} S_j(t)]$$

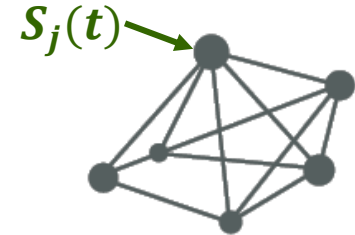
Structural interventions

Removing nodes, adding links, changing weights



Dynamic interventions

External signals $S_j(t)$ to selected nodes



Functional interventions

Manipulating F_i and Q_{ij} or their parameters





Spatio-temporal spreading patterns

A_{ij}

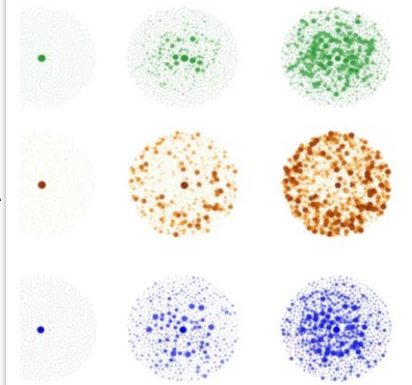


$F_i(x_i, \varphi_i)$

$Q_{ij}(x_i, x_j, \theta_{ij})$



Diverse & unpredictable

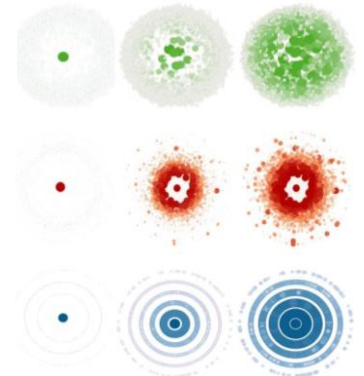


Time

Control parameter θ
Individual node response time

$$\tau_i \sim k_i^\theta$$

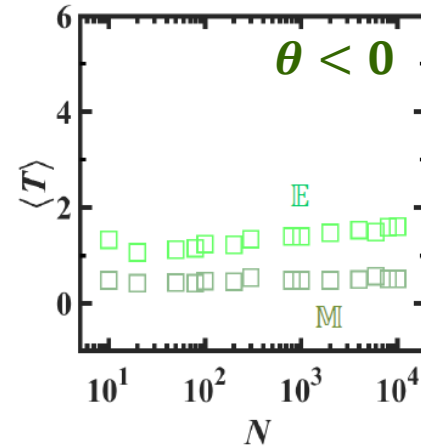
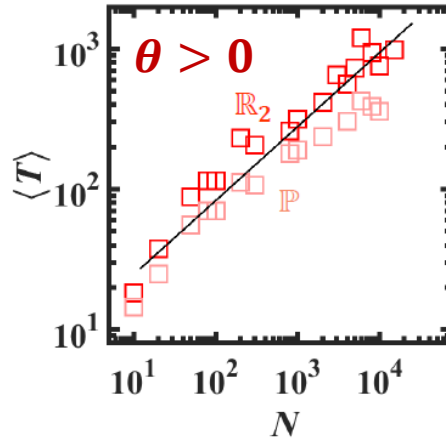
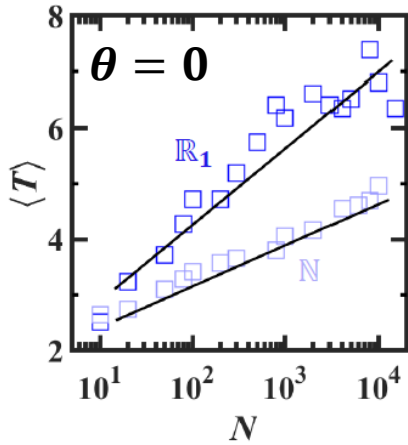
Universal



Time



Soft stability

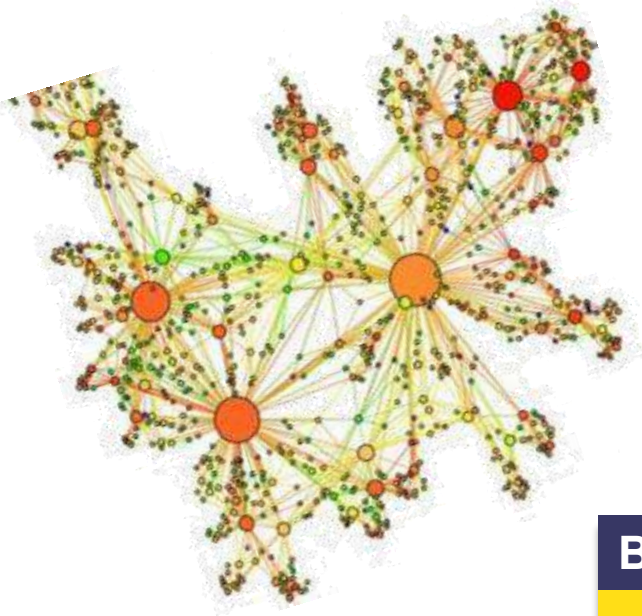


Beyond stability

How much time do we have before an undesired transition occurs

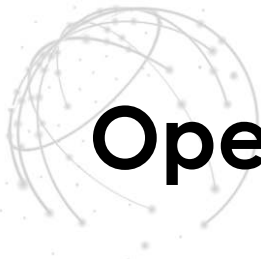


Optimizing functionality vs. resilience



Beyond stability

How much time do we have before an undesired transition occurs



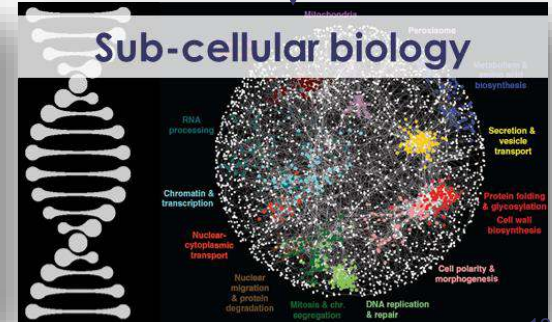
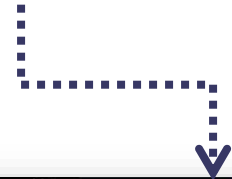
Open threads

Top-down Identify macroscopic control parameters (β, θ)

Bottom-up Selecting nodes for intervention (real-time mitigation)

Stability vs. resilience Enriching the discussion on stability

Functionality vs. resilience Can we introduce balanced incentives



In the top-down approach systems are influenced by means of global control parameters. Quite often these act as boundary conditions for the system dynamics. To identify such control parameters is a challenge on its own. Often they can be derived from the known macroscopic, or system dynamics. As a major conceptual drawback, control parameters usually reflect limitations of stability, rather than of resilience.

- In the bottom-up approach systems are influenced by specifically targeting some of the system elements, e.g. agents in an agent-based model or nodes in a network representation. Again, two different possibilities exist: (i) the agents can be controlled in their internal dynamics, or (ii) the agent interactions can be controlled.

Can we identify general principles for the bottom-up control of socio-economic or ecological systems? How can driver nodes be identified based on data-driven methods?

Can we explain the breakdown of resilience in social organizations as a misallocation of resources? What is the relation between resilience and the natural tendency of systems to maximize their performance?