Entropy and MaxEnt in NonEq Thermodynamics

Massimiliano Esposito

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Math

Markovian master equation (non-autonomous): $$d_t p_i = w_{ij} p_j$$

Mathematical "Entropy Production": $$\dot{\sigma} = w_{ij} p_j \ln \frac{w_{ij} p_j}{w_{ji} p_i} \geq 0$$

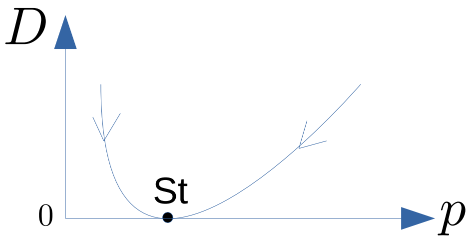
Instantaneous steady state: $$0 = w_{ij} p_i^{st}$$

"Equilibrium": $$w_{ij} p_j^{eq} = w_{ji} p_i^{eq} \quad \forall i,j \longrightarrow \dot{\sigma} = 0$$

$$\dot{\sigma} = \dot{\sigma}_{na} + \dot{\sigma}_a \geq 0 \quad \begin{cases} \dot{\sigma}_{na} = p_i \partial_t (-\ln p_i^{st}) - d_t D(p_i | p_i^{st}) \geq 0 \\ \dot{\sigma}_a = w_{ij} p_j \ln \dfrac{w_{ij} p_j^{st}}{w_{ji} p_i^{st}} \geq 0 \end{cases}$$

relative entropy ≥ 0

Autonomous dynamics:
(min principle)



ME, Harbola, Mukamel, Phys. Rev. E **76**, 031132 (2007)

ME, Van den Broeck, Phys. Rev. E **82**, 011143 (2010)

Physics

Thermodynamics: system exchanging conserved quantities with reservoirs with conjugated intensive variables

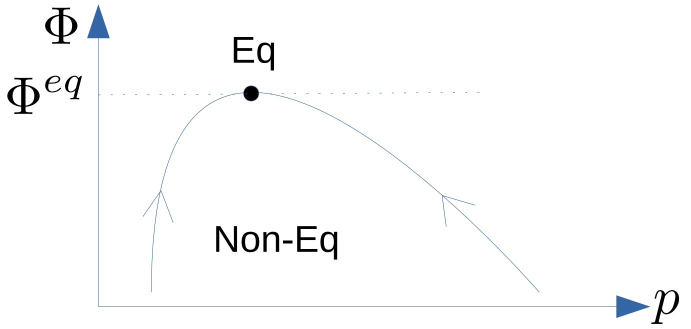
Local detailed balance: $\ln \frac{w_{ij}}{w_{ji}}$ is an entropy change in the reservoirs (and possibly internal)

Physical Entropy Production: $\dot{\sigma} = \partial_t(\Phi^{eq} - \Phi) - d_t D(p|p^{eq}) + A_\alpha J_\alpha \geq 0$

$\Phi^{eq} - \Phi \geq 0$ fluxes-forces

Rao, ME, New J. Phys. **20**, 023007 (2018)
Rao, ME, Entropy **20**, 635 (2018)

Massieu potential e.g. $\Phi = S - \frac{1}{T_0}E + \frac{\mu_0}{T_0}N \cdots$



Autonomous detailed balance dynamics $A_\alpha = 0 \; \forall \alpha$ → Φ is maximized

"Jaynes's maxent is realized dynamically"

In general no thermodynamic max or min principle!

Close to equilibrium (minimum EP principle): $d_t\dot{\sigma} = d_t\dot{\sigma}_{na} \leq 0$

Jiu-Li, Van den Broeck, Nicolis, Z. Phys. B **56**, 165 (1984)
Polettini, ME, Phys. Rev. E **88**, 012112 (2013)