Quantum Jump Approach to Microscopic Heat Engines

Stochastic Thermodynamics of Complex Systems
Online CSH Workshop 2020

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P. Menczel, C. Flindt, KB, arXiv:2005.12231

Outline

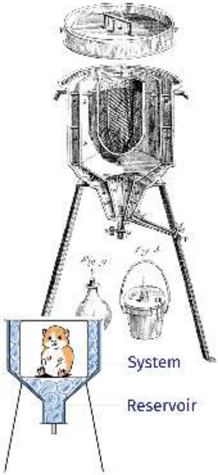
(1) Motivation: Small-Scale Calorimetry

(2) Qubit Engine: Model and Results

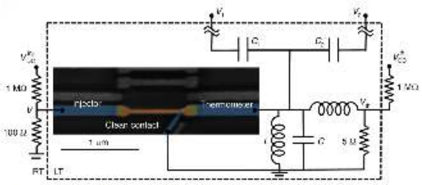
(3) General Picture

Small-Scale Calorimetry

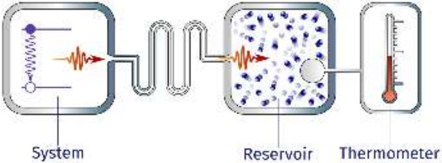
1782 - Ice Calorimeter



2020 - Quantum Calorimeter

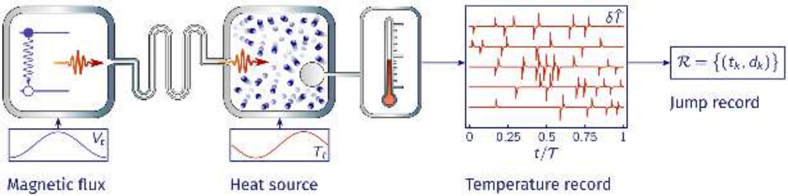


B. Karimi *et al.*, Nat. Commun. **11** 367 (2020)



Qubit Heat Engine - Setup

$$H = -\frac{\hbar\Omega}{2}\left(\Delta\sigma_x + \sqrt{V^2 - \Delta^2}\sigma_z\right)$$



⇒ J. P. Pekola et al., New J. Phys. **15** 115006 (2013)

⇒ M. Campisi, J. P. Pekola, R. Fazio, New J. Phys. **17** 035012 (2015)

→ B. Donvil et al., Phys. Rev. A **97** 052107 (2018)

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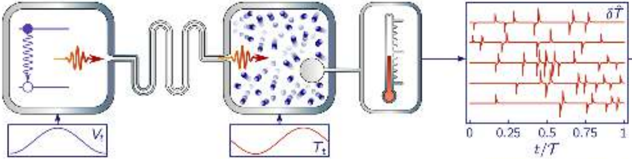


Magnetic flux

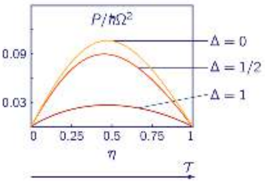
Heat source

Temperature record

$\boxed{\mathcal{R} = \{(t_k, d_k)\}}$

Jump record

Power vs Efficiency



$P = W/\mathcal{T}$

$\eta = W/U \leq 1$

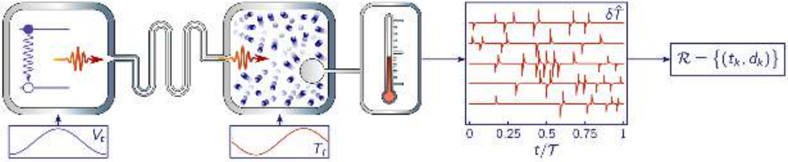
Trade-off relation

$$\boxed{P \leq f[\eta, A_1, A_2, \ldots]}$$

What is f?
What are suitable the parameters A_i?
How to access them?

Qubit Engine: Model and Results

Qubit Heat Engine - Dynamics and Thermodynamics



Master Equation

$$\dot{\rho}_t = -\frac{i}{\hbar}[\rho_t, H_t] + \frac{1}{2}[J_t^+\rho_t, J_t^{+\dagger}] + \frac{1}{2}[J_t^+, \rho_t J_t^{+\dagger}] + \frac{1}{2}[J_t^-\rho_t, J_t^{-\dagger}] + \frac{1}{2}[J_t^-, \rho_t J_t^{-\dagger}]$$

$$H = -\frac{\hbar\Omega}{2}\left(\Delta\sigma_x + \sqrt{V^2 - \Delta^2}\sigma_z\right)$$

$$J^{\pm} = \sqrt{\frac{\mp\kappa\Omega V}{1 - \exp[\pm\hbar\Omega V/T]}}|E^{\pm}\rangle\langle E^{\mp}|$$

First Law

$$Q = W \qquad Q = \int_0^T dt\, \Phi_t \qquad W = \int_0^T dt\, P_t$$

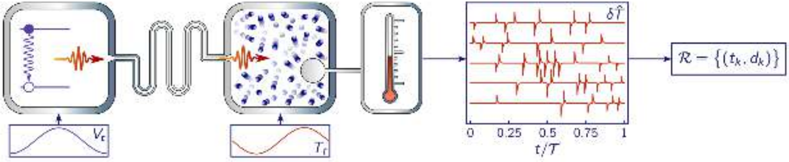
$$\Phi_t = \mathrm{tr}\{\dot{\rho}_t H_t\} \qquad P_t = -\mathrm{tr}\{\rho_t \dot{H}_t\}$$

Second Law

$$\Delta S = (U - W)/T \geq 0 \qquad \Delta S = -\int_0^T dt\, \Phi_t/T_t$$

$$\eta = W/U \leq 1 \qquad U = \int_0^T dt\, (1 - T/T_t)\Phi_t$$

Qubit Heat Engine - Quantum Jumps



Quantum Jump Trajectories

$$|\psi_0\rangle \to |\psi_T\rangle = W[\mathcal{R}]|\psi_0\rangle / \left\| W[\mathcal{R}]|\psi_0\rangle \right\|$$

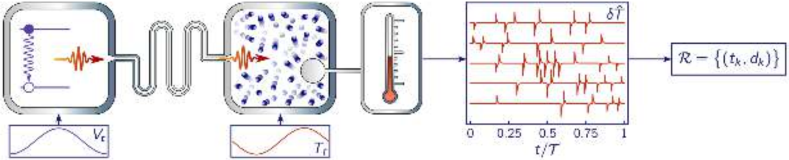
$$W[\mathcal{R}] \equiv W_{T,t_M} \overleftarrow{\prod_{k=1}^{M}} J_{t_k}^{d_k} W_{t_k,t_{k-1}}$$

$$W_{t',t} = \overleftarrow{\exp}\left[-\frac{i}{\hbar} \int_t^{t'} d\tau \, K_\tau \right]$$

$$K_t \equiv H_t - \frac{i\hbar}{2} J_t^{+\dagger} J_t^{+} - \frac{i\hbar}{2} J_t^{-\dagger} J_t^{-}$$

$$p[\mathcal{R}|\psi_0] = \left\| W[\mathcal{R}]|\psi_0\rangle \right\|^2$$

Qubit Heat Engine - Quantum Jumps



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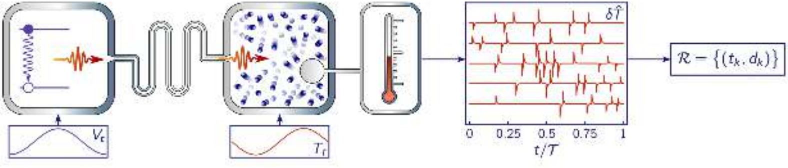
$$p[\mathcal{R}|\psi_0] = \| W[\mathcal{R}]|\psi_0\rangle \|^2$$

Input and Output

$$W = Q = \mathbb{E}\left[\sum_k d_k Q_{t_k}\right] \qquad U = \mathbb{E}\left[\sum_k d_k U_{t_k}\right]$$

$$Q_t = \hbar\Omega V_t \qquad U_t = \hbar\Omega V_t(1 - T/T_t)$$

Qubit Heat Engine - Quantum Jumps



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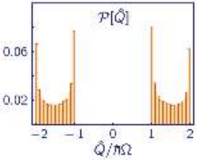
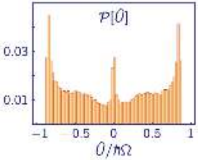
$$p[\mathcal{R}|\psi_0] = \| W[\mathcal{R}]|\psi_0\rangle \|^2$$

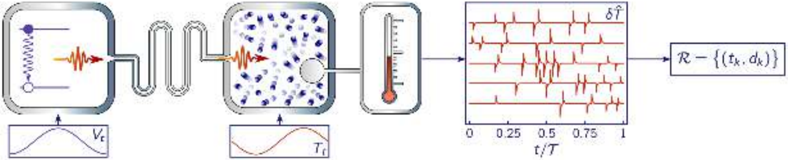
Quantum Jump Distributions

$$\mathcal{P}[\hat{X}] = \frac{1}{\mathcal{A}} \mathbb{E}\left[\sum_k \delta[\hat{X} - d_k X_{t_k}] \right] \qquad \hat{X} = \hat{Q}, \hat{U}$$





$W = Q = \mathcal{A}\langle\hat{Q}\rangle$ $\qquad$ $U = \mathcal{A}\langle\hat{U}\rangle$



Coherence Bound

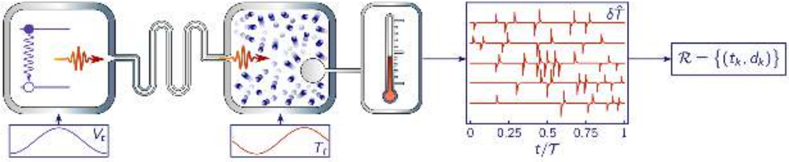
$$\Delta S \geq \Delta S_j = \int_0^{\mathcal{T}} dt\,(j_t^+ - j_t^-)\ln[j_t^+/j_t^-] = \mathcal{A}\langle\hat{\Sigma}_j\rangle$$

$$\Sigma_{jt} = \ln[j_t^+/j_t^-]$$

$$\Delta S_c = \Delta S - \Delta S_j \geq 0$$

$$\Delta S_c = 0 \quad \Leftrightarrow \quad \forall_{0\leq t\leq\mathcal{T}}\ [\rho_t, H_t] = 0$$

Qubit Heat Engine - Beyond the Second Law



Coherence Bound

$$\Delta S \geq \Delta S_{\rm j} = \int_0^T dt\,(j_t^+ - j_t^-)\ln[j_t^+/j_t^-] = \mathcal{A}\langle\hat{\Sigma}_{\rm j}\rangle$$

$$\Sigma_{{\rm j}_t} = \ln[j_t^+/j_t^-]$$

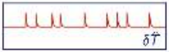
$$\Delta S_{\rm c} = \Delta S - \Delta S_{\rm j} \geq 0$$

$$\Delta S_{\rm c} = 0 \quad \Leftrightarrow \quad \forall_{0\leq t\leq T}\,[\rho_t, H_t] = 0$$

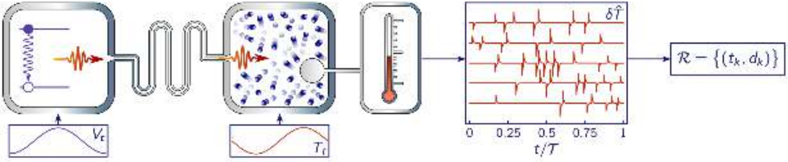
Homogeneity Bound

$$\Delta S_{\rm j} \geq 2\mathcal{A}\lambda_{\hat{X}}\tanh^{-1}[\lambda_{\hat{X}}]$$

$$\lambda_{\hat{X}} = \sqrt{\langle\hat{X}\rangle^2/\langle\hat{X}^2\rangle} \leq 1$$

$\lambda_{\hat{X}} = 1:$ 

$\lambda_{\hat{X}} = 0:$ 



$$\Delta S \geq \Delta S_j \geq 2\mathcal{A}\lambda_{\tilde{X}} \tanh^{-1}[\lambda_{\tilde{X}}]$$

Simple Trade-off Relation

Dissipation rate

Coherence

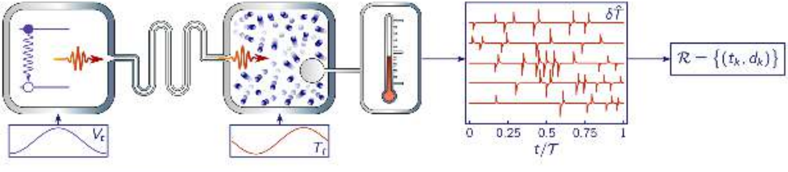
$$P \leq \eta\gamma\sqrt{\langle \hat{U}^2 \rangle} \tanh\left[\frac{\Psi(1-\eta)}{2T}\sqrt{\langle \hat{U}^2 \rangle}\right]$$

Energy scale

$\Psi = \Delta S_j / \Delta S \leq 1$

$\gamma = \mathcal{A}/T$

Qubit Heat Engine - Power vs Efficiency



$$\Delta S \geq \Delta S_i \geq 2\mathcal{A}\lambda_{\tilde{\chi}} \tanh^{-1}[\lambda_{\tilde{\chi}}]$$

Simple Trade-off Relation

Dissipation rate

Coherence

$$P \leq \eta\gamma\sqrt{\langle \hat{O}^2\rangle}\tanh\left[\frac{\Psi(1-\eta)}{2\mathcal{T}}\sqrt{\langle \hat{O}^2\rangle}\right]$$

Energy scale

$\Psi = \Delta S_i / \Delta S \leq 1$

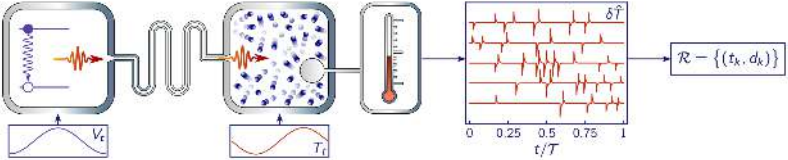
$\gamma = \mathcal{A}/\mathcal{T}$

Optimal Trade-off Relation

$$P \leq \eta\gamma z_\eta \tanh\left[\frac{\Psi(1-\eta)}{2\mathcal{T}} z_\eta\right]$$

$$z_\eta = \frac{\langle \hat{Q}^2\rangle\langle \hat{O}^2\rangle - \langle \hat{R}^2\rangle^2}{\langle \hat{Q}^2\rangle - 2\eta\langle \hat{R}^2\rangle + \eta^2\langle \hat{O}^2\rangle}$$

$R_t = \hbar\Omega V_t\sqrt{\eta_t}$



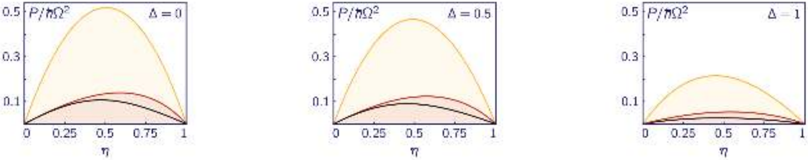
$$H = -\frac{\hbar\Omega}{2}\left(\Delta\sigma_x + \sqrt{V^2 - \Delta^2}\sigma_z\right)$$

Simple Trade-off Relation

$$P \leq \eta\gamma\sqrt{\langle\hat{U}^2\rangle}\tanh\left[\frac{\Psi(1-\eta)}{2T}\sqrt{\langle\hat{U}^2\rangle}\right]$$

Optimal Trade-off Relation

$$P \leq \eta\gamma z_\eta \tanh\left[\frac{\Psi(1-\eta)}{2T} z_\eta\right]$$



The General Picture

The General Picture

Multi-Level Systems + 1 Reservoir

$$\Delta S \geq \Delta S_{\mathrm{j}} \geq 2\mathcal{A}\lambda_{\hat{\chi}} \tanh^{-1}[\lambda_{\hat{\chi}}]$$

$$P \leq \eta\gamma\sqrt{\langle \hat{U}^2\rangle} \tanh\left[\frac{\Psi(1-\eta)}{2T}\sqrt{\langle \hat{U}^2\rangle}\right] \longrightarrow P \leq \eta(1-\eta)\frac{\vartheta_0\Delta_u^2}{T}\int_0^{\mathcal{T}} dt\,(f_t^u)^2 \longrightarrow P \leq \frac{\vartheta_0\Delta_u^2}{4T}\int_0^{\mathcal{T}} dt\,(f_t^u)^2$$

⇾ KB, U. Seifert, PRE **93** 062134 (2016)

$$P \leq \eta\gamma z_\eta \tanh\left[\frac{\Psi(1-\eta)}{2T} z_\eta\right] \longrightarrow P \leq \eta(1-\eta)\frac{\vartheta_0\Delta_u^2}{T}\int_0^{\mathcal{T}} dt\,(f_t^u - \bar{f}^u)^2 \longrightarrow P \leq \frac{\vartheta_0\Delta_u^2}{4T}\int_0^{\mathcal{T}} dt\,(f_t^u - \bar{f}^u)^2$$

⇒ KB, M. Bauer, U. Seifert,
PRL **119** 170602 (2017)

Linear response

$H_t = H + \Delta_w G^w f_t^w$

$T_t = T\,(1 + \Delta_u f_t^u)$

$\Delta_w, \Delta_u \ll 1$

$\Psi \leq 1$

The General Picture

Multi-Level Systems + 1 Reservoir

$$\Delta S \geq \Delta S_j \geq 2\mathcal{A}\lambda_{\dot{X}}\tanh^{-1}[\lambda_{\dot{X}}]$$

$$P \leq \eta\gamma\sqrt{\langle \dot{\mathcal{O}}^2\rangle}\tanh\left[\frac{\Psi(1-\eta)}{2T}\sqrt{\langle \dot{\mathcal{O}}^2\rangle}\right] \rightarrow P \leq \eta(1-\eta)\frac{\vartheta_0\Delta_\nu^2}{T}\int_0^{\mathcal{T}} dt\,(f_t^\nu)^2$$

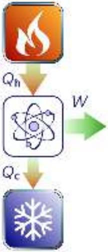
⇒ KB, U. Seifert, PRE **93** 062134 (2016)

$$P \leq \eta\gamma z_\eta \tanh\left[\frac{\Psi(1-\eta)}{2T} z_\eta\right] \rightarrow P \leq \frac{\vartheta_0\Delta_0^2}{4T}\int_0^{\mathcal{T}} dt\,(f_t^\nu - \bar{f}^\nu)^2$$

⇒ KB, M. Bauer, U. Seifert, PRL **119** 170602 (2017)

Multi-Level Systems + N Reservoirs

$$\Delta S \geq \sum_\nu \Delta S_j^\nu \geq \sum_\nu 2\mathcal{A}_\nu\lambda_{\dot{X}}^\nu\tanh^{-1}[\lambda_{\dot{X}}^\nu]$$



$$P \leq \eta_{\rm th}\gamma_{\rm h}\sqrt{\langle \hat{Q}^2\rangle_{\rm h}}\tanh\left[\frac{\Psi(\eta_{\rm C}-\eta_{\rm th})}{2T_{\rm c}}\sqrt{\langle \hat{Q}^2\rangle_{\rm h}}\right]$$

$$\eta_{\rm th} = W/Q_{\rm h}$$

$$\eta_{\rm C} = 1 - T_{\rm c}/T_{\rm h}$$

$$\tanh[x] \leq x$$

$$\Psi < 1$$

$$\rightarrow P \leq \eta_{\rm th}(\eta_{\rm C} - \eta_{\rm th})\Theta/T_{\rm c}$$

$$\Theta = \gamma_{\rm h}\langle \hat{Q}^2\rangle_{\rm h}/2$$

⇒ N. Shiraishi, K. Saito, H. Tasaki, PRL **117** 190601 (2016)

⇒ N. Shiraishi, K. Saito, J. Stat. Phys. **174** 433 (2019)

Multi-Level Systems + 1 Reservoir

$$\Delta S \ge \Delta S_j \ge 2\mathcal{A}\lambda_{\mathcal{X}}\tanh^{-1}[\lambda_{\mathcal{X}}]$$

$$\boxed{P \le \eta\gamma\sqrt{\langle O^2\rangle}\tanh\left[\frac{\Psi(1-\eta)}{2T}\sqrt{\langle O^2\rangle}\right]} \rightarrow \boxed{P \le \eta(1-\eta)\frac{\vartheta_0\Delta_\upsilon^2}{T}\int_0^{\mathcal{T}} dt\,(f_t^\upsilon)^2}$$

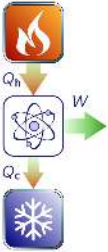
⇒ KB, U. Seifert, PRE **93** 062134 (2016)

$$\boxed{P \le \eta\gamma z_\eta\tanh\left[\frac{\Psi(1-\eta)}{2T}z_\eta\right]} \rightarrow \boxed{P \le \frac{\vartheta_0\Delta_\upsilon^2}{4T}\int_0^{\mathcal{T}} dt\,(f_t^\upsilon-\bar{f}^\upsilon)^2}$$

⇒ KB, M. Bauer, U. Seifert, PRL **119** 170602 (2017)

Multi-Level Systems + N Reservoirs

$$\Delta S \ge \sum\nolimits_\nu \Delta S_j^\nu \ge \sum\nolimits_\nu 2\mathcal{A}_\nu\lambda_{\mathcal{X}}^\nu\tanh^{-1}[\lambda_{\mathcal{X}}^\nu]$$



$$\boxed{P \le \eta_{th}\gamma_h\sqrt{\langle\hat{Q}^2\rangle_h}\tanh\left[\frac{\Psi(\eta_C-\eta_{th})}{2T_c}\sqrt{\langle\hat{Q}^2\rangle_h}\right]} \rightarrow \boxed{P \le \eta_{th}(\eta_C-\eta_{th})\Theta/T_c \qquad \Theta = \gamma_h\langle\hat{Q}^2\rangle_h/2}$$

+ many more

⇒ N. Shiraishi, K. Saito, H. Tasaki, PRL **117** 190601 (2016)

⇒ N. Shiraishi, K. Saito, J. Stat. Phys. **174** 433 (2019)

For: any multi-level systems, number of reservoirs, periodic protocols
Assumptions: weak coupling, detailed balance (instantaneous)