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Dissipation bounds precision and speed

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CSH Online Workshop “Stochastic thermodynamics of complex systems”, May 27-29, 2020

Outline

Part I: *Unifying Thermodynamic Uncertainty Relations,*

Falasco, Esposito, Delvenne, New J. Phys. ***22****, 053046 (2020)*

Part II: *The dissipation-time uncertainty relation*

Falasco, Esposito, [arXiv:2002.03234]

Part I: Thermodynamic Uncertainty Relations

Hilbert uncertainty relation

probability $P(\omega)$ and observable $O(\omega)$ $\in$ Hilbert space H

precision $\mathfrak{p}(O) := \frac{\langle O\rangle^2}{\mathrm{Var}(O)} = \frac{\langle O\rangle^2}{\langle O|O\rangle - \langle O\rangle^2}$

Ritz theorem $\exists\, m \,|\, \forall\, O \;:\; \langle m|O\rangle = \langle O\rangle \;\in$ Dual of H

Cauchy-Schwartz $\mathfrak{p}(O) \leq \langle m\rangle/(1-\langle m\rangle)$

Arbitrary observable: $m = 1 \longrightarrow \mathfrak{p}(O) \leq \infty$

Antisymmetric observables under time reversal $O(\tilde{\omega}) = -O(\omega)$ $\longrightarrow$ $m = \dfrac{p - \tilde{p}}{p + \tilde{p}}$

Tight bound: $\boxed{\mathfrak{p} \leq \dfrac{\langle \tanh \frac{\sigma}{2k_\mathrm{B}} \rangle}{1 - \langle \tanh \frac{\sigma}{2k_\mathrm{B}} \rangle}}$ with $\sigma = k_\mathrm{B} \log \dfrac{p}{\tilde{p}}$

Looser bounds: $\boxed{\mathfrak{p} \leq \dfrac{e^{\langle \sigma \rangle / k_\mathrm{B}} - 1}{2} \qquad \mathfrak{p} \leq N \dfrac{e^{\langle \sigma \rangle / k_\mathrm{B}} - 1}{2} \qquad \mathfrak{p} \leq \dfrac{\langle \sigma \rangle}{2k_\mathrm{B}}}$

General

Markov & N-periodic

Markov & Stationary

Hasegawa, Van Vu, PRL **123**, 110602 (2019)

Potts, Samuelsson, PRE **100**, 052137 (2019)

Barato, Seifert, PRL **114** 158101 (2015)

Gingrich, Horowitz, ... PRL **116** 120601 (2016)
PRE **96** 020103 (2017)

Summary

Unifying framework which:

- pinpoints the mathematical essence of TUR
- gives new (tight) bounds and recovers previous ones
- can be extended to consider other symmetries than time reversal

Falasco, Esposito, Delvenne, New J. Phys. ***22****, 053046 (2020)*

Part II: Dissipation-Time Uncertainty Relation

- Trajectories ω_t in a space Ω_t with probability measure $P(\omega_t)$

- Stopping time $$\tau := \inf\{t \geq 0 : \overset{\overset{\text{Observable}}{\uparrow}}{O}(\omega_t) \in \overset{\overset{\text{Threshold}}{\uparrow}}{D}\}$$

- Space of survived trajectories Ω_t^{s} : If $\omega_t \in \Omega_t^{\mathrm{s}}$ then $O(\omega_t) \notin D$

- Survival probability: $$p^{\mathrm{s}}(t) = \sum_{\omega_t \in \Omega_t^{\mathrm{s}}} P(\omega_t) = \sum_{\omega_t \in \Omega_t} \chi(O(\omega_t))P(\omega_t)$$ $$\searrow \begin{cases} 1 \text{ if } O(\omega_t) \notin D \\ 0 \text{ otherwise} \end{cases}$$

- Instantaneous rate $$r(t) := -\frac{1}{p^{\mathrm{s}}(t)} \frac{dp^{\mathrm{s}}}{dt}(t)$$

- Time reversed trajectory $\tilde{\omega}_t$

- $k_{\mathrm{B}} \log \frac{P(\omega_t)}{P(\tilde{\omega}_t)} =: \int_0^t dt' \dot{\Sigma}(\omega_t)$

Initially in stationary distrib (without threshold)

- Survival probability for $\tilde{O}(\omega) \equiv O(\tilde{\omega}_t)$: $\tilde{p}^{\mathrm{s}}(t) := \sum_{\omega_t \in \Omega_t} \chi(\tilde{O}(\omega_t)) P(\omega_t)$

- Fluctuation Theorem

$$\boxed{p^{\mathrm{s}}(t) = \tilde{p}^{\mathrm{s}}(t) \left\langle e^{-\int_0^t dt' \dot{\Sigma}/k_{\mathrm{B}}} \right\rangle_{\tilde{\mathrm{s}}}}$$

$$\langle F \rangle_{\tilde{\mathrm{s}}} := \sum_{\omega_t \in \tilde{\Omega}_t^{\mathrm{s}}} F(\omega_t) P(\omega_t) / \tilde{p}^{\mathrm{s}}(t)$$

Jensen

$$p^{\mathrm{s}}(t) \geq \tilde{p}^{s}(t) e^{-\int_0^t dt' \langle \dot{\Sigma} \rangle_{\tilde{\mathrm{s}}} / k_{\mathrm{B}}}$$

$$\boxed{\frac{1}{k_{\mathrm{B}}} \langle \dot{\Sigma} \rangle_{\tilde{\mathrm{s}}}(t) \geq r(t) - \tilde{r}(t)}$$

Implicit in work on
“absolute irreversibility”
by Murashita, Funo, Ueda
[Phys. Rev. E **90**, 042110 (2014)]
[New J. Phys. 17 (2015) 075005]
[arXiv:1802.10483]

$$\boxed{\frac{1}{k_{\mathrm{B}}}\langle\dot{\Sigma}\rangle_{\tilde{\mathrm{s}}}(t) \geq r(t) - \tilde{r}(t)}$$

Assume: $r(t) \gg \tilde{r}(t)$ $\quad t \ll \min_t 1/\tilde{r}(t)$ $\longrightarrow$ $\tilde{p}^{\mathrm{s}}(t) \approx 1$ $\quad \langle\dot{\Sigma}\rangle_{\tilde{\mathrm{s}}} \simeq \langle\dot{\Sigma}\rangle = \langle\dot{S}_{\mathrm{e}}\rangle$

$$\boxed{\langle\dot{S}_{\mathrm{e}}\rangle/k_{\mathrm{B}} \geq r(t)}$$

Mean first passage time $\langle\tau\rangle = \int_0^\infty dt\, p^{\mathrm{s}}(t) =: \mathcal{T}$

Dissipation-Time Uncertainty Principle

$$\boxed{\langle\dot{S}_{\mathrm{e}}\rangle\,\mathcal{T} \geq k_{\mathrm{B}}}$$

➡

To realize a process in a time $\mathcal{T}$ at least $k_B/\mathcal{T}$ must be dissipated

Example I: Overdamped particle

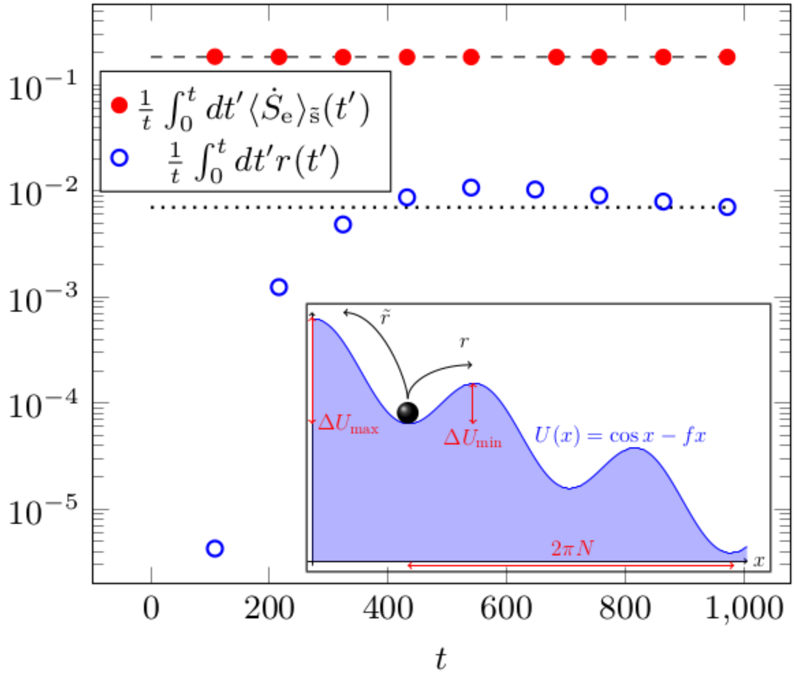
$$\dot{x} = -U'(x) + \sqrt{2/\beta}\xi \qquad x \in [0, 2\pi]$$

$$U(x) = a\cos(x) - fx$$

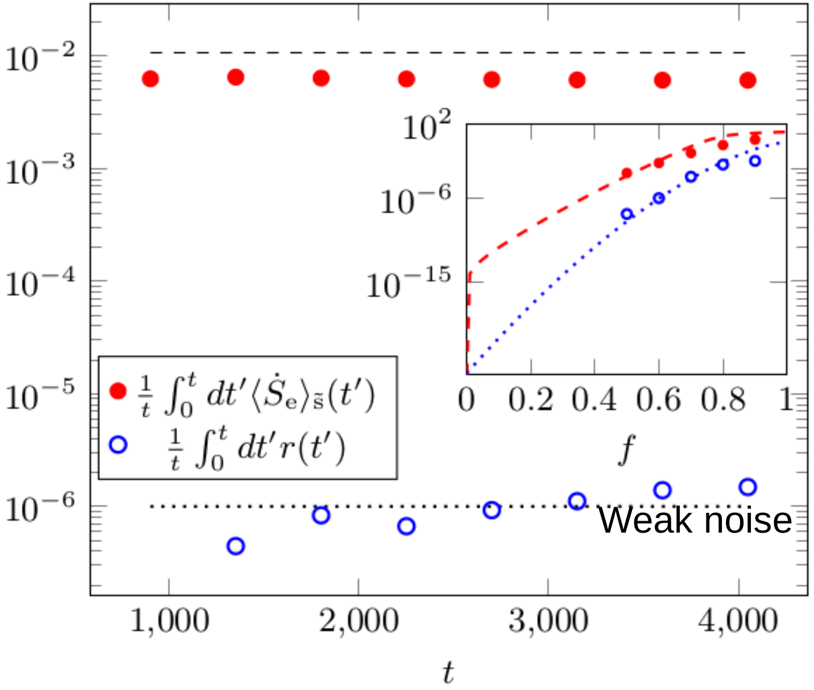
$$O = \int_0^t \dot{x}_{t'}\, dt', \quad D = [2\pi N, \infty)$$

$$\tilde{O} = -2\pi N$$

$N = 11\,, \beta^{-1} = 0.7$ Rare threshold

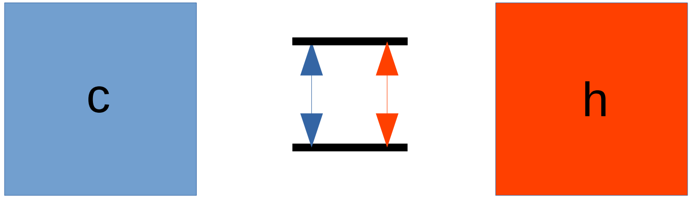


$N = 2\,, \beta^{-1} = 0.07$ Weak noise

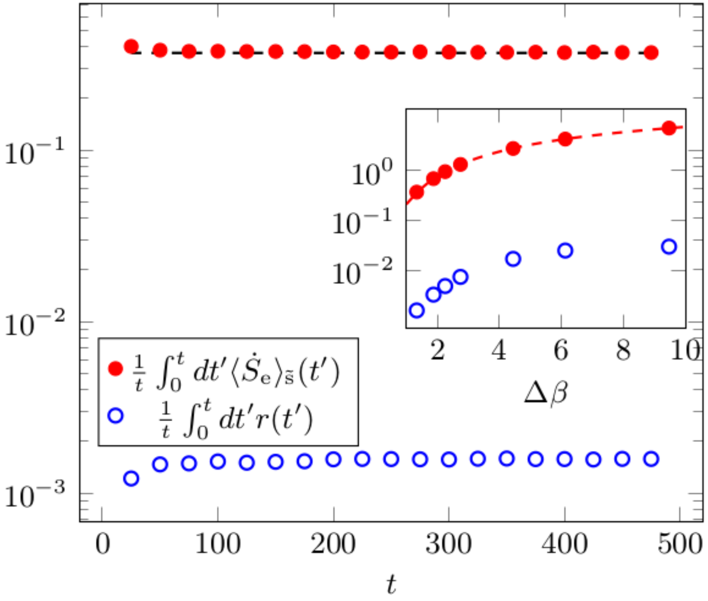


Example II: 2 level system between 2 baths

$$O = (\epsilon_2 - \epsilon_1)\int_0^{\delta} dt' \left[\frac{dN^{\mathrm{c}}_{2\to1}}{dt'} - \frac{dN^{\mathrm{c}}_{1\to2}}{dt'}\right],\ D = [E, \infty)$$



$$w^{\nu}_{i\to j} = e^{-\beta_\nu(\epsilon_j - \epsilon_i)/2}$$



Remarks

- For integrated currents, $\langle \dot{S}_{\mathrm{e}} \rangle \, \mathcal{T} \geq k_{\mathrm{B}}$ can be derived directly from FT

- For integrated currents and stationary Markov dynamics, it can also be obtained from [Roldan, Neri,…, Julicher, PRL **115**, 250602 (2015)]

- Can be generalized to: - periodic driving $\overline{\langle \dot{S}_{\mathrm{e}} \rangle^{\mathrm{B}}} \mathcal{T} \geq k_{\mathrm{B}}$

  - transient driven dynamics $\langle dS/dt \rangle^{\mathrm{B}} + \langle \dot{S}_{\mathrm{e}} \rangle^{\mathrm{B}} \geq k_B r_{\mathrm{F}}(t)$

- Dynamics other than time reversed can be considered

- Contrary to usual speed limits (changes from state to state) the Dissipation-Time UR deals with stationary processes and involves only EP

Conclusion

Thermodynamics constrains nonequilibria

Dissipation bounds precision and speed

Precision	$\frac{\langle O\rangle^2}{\mathrm{Var}(O)} \leq \frac{\langle S_e\rangle}{2k_\mathrm{B}}$	*New J. Phys.* ***22**, 053046 (2020)*
Speed	$\langle \dot{S}_\mathrm{e}\rangle \, \mathcal{T} \geq k_\mathrm{B}$	arXiv:2002.03234

Caution: It does not mean that more dissipation will necessary speed up and improve precision